Exponential Diophantine equations and linear forms in p-adic logarithms

Noriko HIRATA-KOHNO
Department of Mathematics, College of Science and Technology,
Nihon University, Tokyo 101-8305, Japan

We first recall effective Diophantine approximations due to Gel'fond around linear forms in p-adic logarithms and several applications.

We then consider the following equation. Let A, B, C be rationals with $ABC \neq 0$ and $a_1, a_2, \cdots, a_\ell,\ b_1, b_2, \cdots, b_m,\ c_1, c_2, \cdots, c_n$ be rational integers. Suppose the 3 numbers $a_1 a_2 \cdots a_\ell$, $b_1 b_2 \cdots b_m$, $c_1 c_2 \cdots c_n$ are relatively co-prime. Consider $Aa_1^{x_1} a_2^{x_2} \cdots a_\ell^{x_\ell} + Bb_1^{y_1} b_2^{y_2} \cdots b_m^{y_m} + Cc_1^{z_1} c_2^{z_2} \cdots c_n^{z_n} = 0$ where $x_1, x_2, \cdots, x_\ell$, $y_1, y_2, \cdots, y_m,\ z_1, z_2, \cdots, z_n$ are viewed as unknowns in integers.

Gel'fond proved that this equation has only finitely many solutions with an implicit effective upper bound for the height of solutions. We discuss an alternative approach to determine such a bound. We present a result concerning with a system of congruence from which we deduce this finiteness.